## EFFECT OF ELASTIC ANISOTROPY ON THE DISTRIBUTION

## OF STRESSES AT THE TIP OFA CRACK SUBJECTED

TO ELASTIC WAVES
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By the method of dynamic photoelasticity, the effect of elastic anisotropy on the distribution of stresses in the vicinity of a stationary crack is investigated in the paper in the case of action of stress waves. It is shown that in crystals of lithium fluoride the formation of a stress state does not substantially differ from that occurring in an isotropic medium. As a result of an investigation of the dislocation structure at the tip of a crack before and after loading, it is established that the plastic strain at its issue develops by means : of sliding.

Under dynamic loading the trajectory of development of cracks is determined by the stress state that arises at their tips as a result of the action of stress waves. The publications [1-4] are devoted to the study of stress distribution in the vicinity of the crack tip in the homogeneous isotropic material Plexiglas. Since a majority of materials have a complex structural pattern, investigations into the formation of the stress state and the development of cracks in anisotropic media are of theoretical and practical interest. The process of fracture of such materials cannot be comprehended without studying the development of cracks in a single crystal, i.e., when the fracture takes place within the limits of a single grain.

The present work is devoted to an experimental study of these problems. The investigations were carried out on testpieces of lithium fluoride which were cut out along the $\{100\}$ cleavage planes with the dimensions $100 \times 50 \times 5 \mathrm{~mm}^{3}$. The cracks were generated by a slight tap with a knife in the [100] direction. To keep it in a definite zone, this portion of the testpiece was compressed beforehand. The testpieces thus prepared were loaded by means of a microexplosion with a duration of $17-20 \mu \mathrm{sec}$. The stress wave being excited was set to the crack tip at angles of $0,45,90,135$, and $180^{\circ}$.

The filming of the diffraction process was carried out at a speed of $2.5 \cdot 10^{6}$ frames per second in polarized light. On the cinegrams of Fig. 1 we have shown the interaction of the stress wave and the crack for orientations of the latter relative to the direction of propagation of the wave; $\gamma$ is the angle of incidence of the wave in the plane of the crack. When the wave diffracts on the crack, a dynamic stress field arises in its vicinity; under certain conditions it is capable of stimulating the growth of the crack. The development of the crack under the action of the wave takes place along the cleavage planes. This is explained, on one hand, by the fact that in these planes fracture takes place with a minimum energy expenditure, and on the other hand, as will be shown below, by the fact that, for certain angles of incidence of the wave, the stress state arising in the case of diffraction is such that it favors the development of the crack only in this direction.

When the wave propagates along one of the sides of the crack, i.e., when the angle of incidence is zero, a new crack grows into the shade region at an angle of $90^{\circ}$ to the original crack. These results coincide with the data obtained in [1] on an isotropic material. However, the photoelastic patterns of stress distribution being observed cannot be compared, since in an anisotropic material the axes of the optical ellipsoid and the stress ellipsoid do not coincide. Therefore, lines of equal intensity corresponding to the same differences of the passage of rays do not correspond to the lines of equal maximum shear stresses, as is the case for an isotropic medium.

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## TABLE 1

| $J$ | $0^{\circ}$ |  |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 0 | 22.5 | 45 | 60 | 90 | 120 | 133 | 157.5 | 180 |
| 150 | 2.8 | 2.3 | 1.0 | 0.8 | 0.7 | 0.8 | 1.7 | 3.0 | 3.3 |
| 130 | - | 4.5 | 1.7 | 1.4 | 1.5 | 1.6 | 3.0 | 4.0 | 4.5 |
| 100 | - | - | 2.7 | 1.8 | 1.9 | 2.6 | 5.0 |  | - |
| 80 | - | - | 4.4 | 3.1 | 3.4 | 5.0 | 6.3 | - | - |
| 60 |  | - |  | 5.2 | 7.0 | 8.4 |  |  | - |



Fig. 1


Fig. 2

To find the distribution of maximum shear stresses, we must make a transition from the intensity distribution being observed to stress quantities. Such a transition can be made using the results of $[5,6]$. According to [5], the difference of the passage of rays is determined as follows:

$$
\begin{equation*}
\delta=d_{0}\left(B_{1}-B_{2}\right)\left(\sigma_{x}^{*}-\sigma_{\mathcal{y}}^{*}\right) \tag{1}
\end{equation*}
$$

where $\sigma_{\mathrm{x}}{ }^{*}, \sigma_{\mathrm{y}}{ }^{*}$ are normal stresses averaged across the thickness of the testpiece, applied to planes that are perpendicular to the axes of the optical ellipsoid; $d_{0}$ is the thickness of the testpiece

$$
\begin{equation*}
\delta_{x^{*}}-\sigma_{u^{*}}=\left(\sigma_{1}-\sigma_{2}\right) \cos 2 x \tag{2}
\end{equation*}
$$

The intensity of light passed through a circular polariscope is given by the expression

$$
\begin{align*}
& J=J_{\max } \sin a^{* / 2}  \tag{3}\\
& \alpha^{*} / 2 \pi=\delta / \lambda \tag{4}
\end{align*}
$$

Using the relationships (1)-(4), we can express $\tau_{\max }$ as follows:

$$
\begin{align*}
& \tau_{\operatorname{rnax}}=\frac{\lambda}{2 \pi d_{0}\left(B_{1}-B_{2}\right) \cos 2 \alpha} \arcsin \sqrt{\frac{J^{*}-J_{0}{ }^{*}}{J_{\max }^{*}}}  \tag{5}\\
& \left(B_{1}-B_{2}\right) \cos 2 x=\frac{A B\left(B \cos ^{2} 2 \beta+A \sin ^{2} 2 \beta\right)}{B^{2} \cos ^{2} 2 \beta+\Lambda^{2} \sin ^{2} 2 \beta} \tag{6}
\end{align*}
$$

where $\beta$ is the angle between the [100] direction and the principal axis of the optical ellipsoid, $\mathrm{J}^{*}$ is darkening on the film, $J_{0}{ }^{*}$ is darkening of the background of the film, and $J^{*} \max$ is the maximum darkening of the film in the case of parallel polarizers after subtraction of $J_{0}{ }^{*}$.


Fig. 3

The values of the necessary constants have been taken from [5], where they were obtained in static tests on this material. The applicability of such an approach is explained by the fact that in the given work we investigate the character of stress distribution at the origin of the crack and not the absolute values of stresses.

Knowing $\mathrm{d}_{0}, \mathrm{~J}^{*}, \mathrm{~J}_{0}{ }^{*}, \mathrm{~J}^{*}$ max and $\beta$, we can quantitatively determine $\tau_{\max }$ at any point of the field being investigated. The intensity of darkening of the film was measured by means of an MF-2 microlight meter. The position of the isoclines was determined experimental ly by high-speed filming in a plane polariscope. Investigations showed that in the case of diffraction of the wave on the crack the stress state in its vicinity varied only in magnitude; as a result the position of isoclines did not vary with time. This can be shown also analytically. In [4] expressions are produced for $\sigma_{x}, \sigma_{y}$, and $\tau_{\mathrm{xy}}$ in the case of a unit action. In this case $\beta$, the isocline parameter, is determined according to the expression

$$
\beta=\operatorname{arctg} 2 \tau_{x y} /\left(\sigma_{x}-\sigma_{y}\right)
$$

For an arbitrary load

$$
\sigma^{A}=A \sigma^{1}, \quad \beta^{A}=\operatorname{arctg} 2 \tau_{x y} /\left(\sigma_{x}^{A}-\sigma_{y}^{A}\right)=\beta
$$

The results of light-metering the cinegram for the case where the wave propagates along one side of the crack, $\gamma=0$, are presented in Table 1.

In Table 1 we have presented the distances in millimeters from the crack tip to the line of equal intensity for various angles $\theta$ ( $\theta$ is the angle between the plane of crack and a chosen direction).

Using the results concerned with the determination of the position of isoclines for various angles of the polarizer and the analyzer, we have plotted the distribution of maximum shear stresses at the crack tip. In Fig. 2 we have presented the distribution of maximum shear stresses for the case $\gamma=0$. We see that the gradient of the splitting stresses is directed along the normal to the plane of the crack. The development of the crack takes place in the same direction. Comparing the results thus obtained with the data of [1], we can draw the conclusion that in the anisotropic material LiF the stress distribution at the tip of a stationary crack, under the action of stress waves, does not differ from the distribution in an isotropic medium. This is confirmed in the case of angles of incidence of the wave of 90,135 , and $180^{\circ}$. The pattern of shear stress distribution shows that the isochromatics represent practically semicircles whose centers are located on the line of the crack, while the isoclines represent radial lines converging to the crack tip. Before the neutral plane (the plane of gradient of shear stresses) a region of compressive stresses is formed, while behind it a region of tensile stresses is formed.

An increase in the angle of incidence of the wave is accompanied by a reduction in the stress concentration at the crack tip. Depending on the angle of attack, the concentration factor varies from 1 to 3 . This signifies that the strain directed along the crack is responsible for the formation of the stress state. As the angle of incidence of the wave increases, the component of strain along the crack decreases; as a result, the stress state at the crack tip decreases.

An analogous polarized light pattern is formed when a concentrated static load is applied along the edge of the plate [7]. If we use the results [7], then it is not difficult to determine $\sigma_{r}$ and find approximately the distribution of $\sigma_{\mathrm{x}}, \sigma_{\mathrm{y}}$, and $\tau_{\mathrm{xy}}$ at the crack tip.

Consideration of the variation of the dislocation structure at the crack tip in the case of wave action is of considerable interest. For this, one of the sets of testpieces was subjected to annealing at a temperature of $740^{\circ} \mathrm{C}$ for 24 h with subsequent slow cooling at the rate $5 \% \mathrm{~h}$. Then, using the method of selective etching, investigation of the dislocation structure at the crack tip was carried out before and after loading. In Fig. 3 we have shown the variation of the dislocation sturcture: an overall increase in the density of dislocations over the workpiece from $3.18 \cdot 10^{4}$ to $4.55 \cdot 10^{4} \mathrm{~cm}^{2}$. Certain particular features appear here: in the direction of the gradient of tangential stresses formation of slip lines takes place; these propagate over
a distance up to 1 mm . This points to the fact that in the given case the direction of plastic strain of the crystal takes place by means of sliding.

Investigations of the dislocation structure also confirm the fact that under the conditions of dynamic loading the plastic strain preceding the fracture is localized in a small volume. If we take into account the fact that the growing of the crack takes place for a considerable intensity of the wave, when the crack cannot develop a high growth rate, then the localization of the plastic strain can be explained only by the form of loading.

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